

ABSTRACTS

Some examples of Randers spaces

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Let $(\widetilde{M}, \widetilde{g}, \widetilde{P})$ be an Riemann almost product manifold and M a hypersurface of it.

Let g be the Riemann metric on M induced by \widetilde{g} and let N be an unitary vector field normal to M . We decompose $\widetilde{P}N$ as $\widetilde{P}N = \xi + aN$, where ξ is a vector field tangent to M and a is a smooth function on M . We consider the 1-form $b(X) = g(\xi, X)$ for any vector field X tangent to M . Assume that the immersion $M \hookrightarrow \widetilde{M}$ is such that $\xi \neq 0$ and $a \neq 0$. We call *generic* such an immersion. Our first result is that the pair (g, b) provides a strictly convex Randers metric on M i.e. $\|b\| < 1$, where $\| \cdot \|$ is taken with respect to g .

For any vector field X tangent to M , we decompose $\widetilde{P}X$ in the form $\widetilde{P}X = PX + u(X)N$. It comes out that u coincides with b . Let A be the Weingarten operator of the $M \hookrightarrow \widetilde{M}$. We prove:

- (i) the 1-form b is closed if and only if $AP = PA$,
- (ii) the Randers structure (g, b) reduces to a Berwald structure if and only if $PA = aA$,
- (iii) the vector field ξ is Killing if and only if $PA + AP = 2aA$.

We explicitly wrote the Randers metrics provided by (g, b) for surfaces in E^3 with $\widetilde{P}(x^1, x^2, x^3) = (x^1, x^2, -x^3)$ or $(x^1, -x^2, -x^3)$, for the sphere S^3 in E^4 with $\widetilde{P}(x^1, x^2, x^3, x^4) = (x^3, x^4, x^1, x^2)$ and we explicitly computed some Finslerian invariants of these Randers metrics.

On the Weyl tensor depending on the position only

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In a Finsler space the components of a tensor field are usually functions of position and direction. At first M. Matsumoto and S. Bcs considered Finsler spaces having h-curvature tensor whose components are functions of position alone. (Publ. Math., Debrecen, 55(1999), 199–210; 57(2000), 185–192) Z. Shen showed: such compact Finsler spaces must be Landsbergian. (Publ. Math., Debrecen, 58(2001), 263–274). The aim of the present lecture to answer the following question: What happens when the Weyl tensor depends only on position? (Z. Shen: Some open problems in Finsler geometry, Problem 46, <http://www.math.iupui.edu/~zshen/Research/preprintindex.html>)

Capacity of compact sets and a class of Finsler manifolds

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The existence of a distance function on a Finsler manifold, using the capacity of a compact set, is discussed. This distance function divides Finsler manifolds into two classes, denoted in this talk by I and II. We want to show that the topology generated by this distance on the Finsler manifolds of class I, coincides with the intrinsic topology of the manifold.

On invariant and anti-invariant submanifolds of unit tangent bundles generated by unit vector fields

T. Q. Binh, L. Vanhecke and E. Boeckx

Let (M, g) be a Riemannian manifold and (T_1M, G_s) its unit tangent sphere bundle equipped with the Sasakian metric. Any smooth unit vector field ξ

on M determines an embedding of M into (T_1M, G_s) . If M is compact and orientable, we can define the volume of ξ as the volume of the corresponding embedded manifold. In this way we get the so-called volume functional on the set of unit vector fields on M . Then, we say that a unit vector field is minimal if it is a critical point of the volume functional. In similar way we get the notion of harmonic vector field as a critical point of energy functional if unit vector field is considered as a mapping between the Riemannian manifolds (M, g) and (T_1M, G_s) .

On the other hand, it is well-known that the unit tangent bundle (T_1M, G_s) admits the contact metric structure, thus the unit vector field ξ can be thought as an isometric embedding of a Riemannian manifold (M, ξ^*G_s) into the contact metric Riemannian manifold (T_1M, G_s) . Thus, (M, ξ^*G_s) is a submanifold of (T_1M, G_s) . In analogy with the notions of harmonic or minimal vector fields, we say that a unit vector ξ is invariant (or anti-invariant) if (M, ξ^*G_s) is invariant (or anti-invariant respectively) submanifold of the contact metric manifold (T_1M, G_s) . The aim of the present paper is to classify the invariant and the anti-invariant vector fields on a Riemannian manifold (M, g) . Since invariant submanifolds are also minimal in contact metric manifolds, we get new proofs for some results of González-Dávila and Vanhecke [Go-Va].

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Positively curved complex Finsler manifolds

Tran Quoc Binh and Ioan Radu Peter

In the talk we present a compactness theorem for positively curved pseudo-Kähler Finsler manifolds. We also touch some coincidence results in this setting.

A special nonlinear connection in higher order geometry

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Given a nonlinear connection on the tangent bundle TM of a differentiable manifold M , we show that the equations of deviations of its autoparallel curves determine a nonlinear connection on $Osc^2M = J_0^2M$. In particular, Jacobi equations in Finsler spaces determine such a nonlinear connection.

We also extend the above construction for the solutions of some systems of higher order ordinary differential equations.

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Canonical semispray and nonlinear connection of a second order Lagrange space

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The canonical semispray of a second order regular Lagrangian is uniquely determined by two associated Cartan-Poincaré one-forms. This third order vector field induces a nonlinear connection that is a Lagrangian subbundle for the presymplectic structure. With respect to this nonlinear connection, the second order dynamical derivative of the metric tensor vanishes.

Randers metrics of scalar flag curvature

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We study an important class of Finsler metrics — Randers metrics. We classify Randers metrics of scalar flag curvature whose S-curvatures are isotropic. This class of Randers metrics contains all projectively flat Randers metrics with isotropic S-curvature and Randers metrics of constant flag curvature.

Our main theorem is as follows

Theorem 0.1 *Let $F = \alpha + \beta$ be a Randers metric on a manifold M of dimension $n \geq 3$, which is expressed in terms of a Riemannian metric h and a vector field W by*

$$h\left(x, \frac{y}{F} - W_x\right) = 1.$$

Then F is of scalar flag curvature $\mathbf{K} = K(x, y)$ and of isotropic S-curvature $\mathbf{S} = (n + 1)c(x)F$ if and only if at any point, there is a local coordinate system in which h , c and W are given by

$$h = \frac{\sqrt{|y|^2 + \mu(|x|^2|y|^2 - \langle x, y \rangle^2)}}{1 + \mu|x|^2},$$

$$c = \frac{\delta + \langle a, x \rangle}{\sqrt{1 + \mu|x|^2}}$$

$$W = -2\left\{ \left(\delta\sqrt{1 + \mu|x|^2} + \langle a, x \rangle \right) x - \frac{|x|^2 a}{\sqrt{1 + \mu|x|^2} + 1} \right\} + xQ + b + \mu\langle b, x \rangle x,$$

where δ, μ are constants, $Q = (q_j^i)$ is an anti-symmetric matrix and $a, b \in \mathbf{R}^n$ are constant vectors. In this case, the flag curvature is given by

$$\mathbf{K} = \frac{3c_x^m y^m}{F} + \sigma,$$

where $\sigma = \mu - c^2 - 2c_x^m W^m$.

Generalized n-ary composition laws in the algebra H(4) and associated metric forms

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The paper deals with the problem of multilinearization of Algebra H(4) norms. New binary, ternary and quaternary operations in the algebra H(4) with isotropic basis are defined (Zassenhaus multiplication). The proof is provided that quadratic Minkowski norm of the element from algebra H(4), Berwald-Moore norm, associated with the quadric form, as well as the norm associated with the cubic form are equal to the values of the introduced binary, quaternary and ternary operations when operation arguments (Zassenhaus multiplicands) are the same.

The dynamical systems of rheonomic Finslerian mechanical systems

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The geometric study of dynamical systems is an important section of contemporary mathematics due to the applications in Mechanics, Theoretical Physics, Control Systems, Economy and Biology. If M is a differentiable manifold that corresponds to the configuration space, a dynamical system can be locally given

by a system of ordinary differential equations of the form $\dot{x}^i = f^i(t, x)$, which are called equations of evolution. The dynamical system of a Lagrangian mechanical system cannot be correctly defined without geometrical frameworks of the phase manifold TM .

Paper [7] studied the Finslerian mechanical systems, their equations and their associated dynamical systems.

In this paper, one defines the notion of rheonomic Finslerian mechanical system $\Sigma = (M, F(x, y), F_i(x, y, t))$, where $RF^n = (M, F(x, y, t))$ is a rheonomic Finsler space and $F_i(x, y, t)$ are the external forces and will be studied the dynamical system of a rheonomic Finslerian mechanical system. If F_i are the components of a globally defined d -covector field on $TM \times R$, then one can associate to the mechanical system a globally defined vector field S on $TM \times R$, which will be called the canonical semispray of the mechanical system. The geometry of the mechanical system is the geometry of the semispray S . All geometric objects one can derive from S , such as a nonlinear connection N , a N -linear connection, a metrical N -linear connection will use to study the mechanical system.

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Field theory and Finsler spaces

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We suggest to construct Lagrangian of a field (or fields), using only a metric function of Finsler space. Namely, as one divided by volume, which included

a unit vector, running all points of indicatrix in a tangential space, if the tangential space is Euclidean that. For a space conformally connected with the Minkowski space we obtained cosmological equation, in the case of exponential dependence on time and spherically symmetric dependence on coordinates. It follows from the equation that the Hubble law is fulfilled, when distances from origin of coordinates are considerably larger than University sizes. The cosmological equation was written for the field, describing the University with geometry conformally connected with geometry of Polynumbers H-4, which possesses the Berwald-Moore metric.

A framed $f(3, 1)$ -structure on tangent manifolds

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A tangent manifold is a pair (M, J) with J a tangent structure ($J^2 = 0$, $\ker J = \text{im } J$) on the manifold M . A systematic study of tangent manifolds was done by I. Vaisman in [3]. One denotes by HM any complement of $\text{im } J := TV$. Using the projections h and v on the two terms in the decomposition $TM = HM \oplus TV$ one naturally defines an almost complex structure F on M . Adding to the pair (M, J) a Riemannian metric g in the bundle TV one obtains what we call a GL -tangent manifold. We assume that the GL -tangent manifold (M, J, g) is of bundle-type, that is M possesses a globally defined Euler or Liouville vector field. This data allow us to deform F to a framed $f(3, 1)$ -structure \mathcal{F} . The later kind of structures have origin in a paper by K. Yano. Then we show that \mathcal{F} restricted to a submanifold that is similar to the indicatrix bundle in Finsler geometry, provides a Riemannian almost contact structure on the said submanifold.

The present results extend to the framework of tangent manifolds our previous results on framed structures of the tangent bundles of Finsler or Lagrange manifolds, see [1], [2].

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Submanifolds in Riemannian manifold with golden structure

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We consider a m -dimensional manifold \widetilde{M} ($m \in \mathbf{N}^*$), endowed with a Riemannian metric \widetilde{g} and a golden structure \widetilde{P} such that $\widetilde{g}(\widetilde{P}U, V) = \widetilde{g}(U, \widetilde{P}V)$ for every tangent vector fields $U, V \in \chi(\widetilde{M})$. An $(1,1)$ -tensor field \widetilde{P} on \widetilde{M} is a golden structure if it satisfies the equation $\widetilde{P}^2 = \widetilde{P} + Id$, where Id is the identity on \widetilde{M} . The name of the golden structure is suggested by its equation, which is the same as that satisfied by the golden number (i.e. $x^2 = x + 1$). We remark that one of the eigenvalues of the \widetilde{P} is the golden number, which is noted by ϕ . We show that the pair $(\widetilde{P}, \widetilde{g})$ induces on any submanifold M of codimension r in \widetilde{M} ($r \in \mathbf{N}^*$, $r < m$), a structure denoted by $(P, g, \xi_\alpha, u_\alpha, (a_{\alpha\beta})_r)$ (with $\alpha, \beta \in \{1, \dots, r\}$), where P is an $(1,1)$ -tensor field, ξ_α are tangent vector fields on submanifold M , u_α are 1-forms on M and $(a_{\alpha\beta})_r$ is a $r \times r$ matrix of real functions on M . We give some properties of these structures. Examples on spheres of codimension 1, 2 or 3 in an Euclidian space are constructed.

Key words: submanifolds, Riemannian manifold, induced structure.

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Finsler foliations

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In their recent paper A. Miernowski and W. Mózgawa (cf. A. Miernowski, W. Mózgawa, Lift of the Finsler foliation to its normal bundle, Diff. Geom. Appl. **24** (2006), 209–214) introduce a Finslerian foliation as a foliation defined by a cocycle modelled on a Finsler manifold such that the coherence transformations are local isometries of the model Finsler manifold. At the same time they pose a question whether any Finslerian foliation is Riemannian, which is a particular case of the problem presented by E. Ghys. A. Miernowski and W. Mózgawa gave a partial answer to the question, namely they prove that the induced foliation of the normal bundle of a Finslerian foliation is Riemannian. Prof. R. Wolak and myself, we go one step further and give the positive answer to this question in the case of a compact manifold. The main result is the following theorem.

Theorem. *Let F be a Finslerian foliation of a compact manifold M . Then F is a Riemannian foliation.*

Lagrange–Finsler geometry and mechanics

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The geometric study of dynamical systems on the differentiable manifolds is an important chapter of contemporary mathematics due its applications in Theoretical Physics, Mechanics, Control Systems etc. Applied in Mechanics we are obliged to study these systems on the phase space. So we can consider the extension of classical mechanics for the case of Finslerian or, more general, Lagrangian mechanical systems. Thus, the geometrical tool adequate is that of the Lagrange geometry or Hamiltonian geometry, introduced by R. Miron and expound in the books, [5], [6].

In the present paper we define and briefly present the geometrical theory of the Lagrangian and Finslerian Mechanical system establishing: the canonical semispray, evolution equations, canonical nonlinear connections, N - metrical connections, them structure equations and the almost Hermitian model.

Symmetric and Ricci symmetric Finsler spaces with concircular transformations

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Concircular transformations introduced by K. Yano in Riemannian geometry; and later, extended by K. Takano to affine geometry with recurrent curvature were further studied by M. Okumura in different types of Riemannian Manifolds. K. Takano also discussed various alternative forms for the covariant derivative of the generator of and infinitesimal transformation which were; later, extended by R. B. Misra et al. to Finslerian manifolds of recurrent curvature; particular cases of these transformations such as Contra, Concurrent and special Concircular transformations defining projective motions were also studied by R. B. Misra in a recurrent Finsler Manifolds. Presently, we discuss the existence of a concircular infinitesimal transformation of a Symmetric and Ricci-symmetric Finsler manifolds.

Key words: Symmetric Finsler manifolds, Ricci-symmetric Finsler manifolds, projective motion, affine motion.

On complex Finsler spaces with Randers metric

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In this paper we introduce in study a new class of complex Finsler spaces, namely the complex Randers spaces. We describe the complex Randers metrics as the solutions of Zermelo's problem of navigation on Hermitian manifolds. Based on it, we construct such examples of complex Randers metrics and we study some of their geometrical properties. Using the length arc parametrization for the extremal curves of the Euler-Lagrange equations we obtain a complex nonlinear connections of Lorentz type in a complex Randers space.

The Euler-Lagrange PDE and Finsler metrizable

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We investigate the following question: under what conditions can a second-order homogeneous ordinary differential equation (spray) be the geodesic equation of a Finsler space.

Finsler manifolds with vanishing h -curvature

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In this talk we shall introduce the notion of dual connection of Finsler connections, and we show the following theorem as an application of it.

Theorem. *Let (M, L) be a Finsler manifold. If the h -curvature R_D^{HH} of the Berwald connection D in (M, L) vanishes identically, then M admits a Hessian structure (\underline{D}, g) .*

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Geometric structures associated to some systems of first order partial differential equations

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In the paper we consider systems of first order partial differential equations, to which we associate variational forms and adjoint forms. The coefficients of these forms lead us to some geometric structures. We obtain distinguished tensors, generalized non-linear connection.

Polymetric geometry

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During the period 1935 - 1936 P.K Rashevskii made a number of reports at the Kogan's seminar "Vector and tensor analysis". He came to the conclusion that polymetric geometries exist. The main idea of such geometries is appearing of duality, when change of a straight line and a point leads to 2-D Riemann geometry. The duality is observed for 2-D quadratic spaces, namely, for elliptic and hyperbolic flats. There are no in Riemann spaces. From symmetry sense Rashevskii would like to see a possibility of change a flat and a n-flat on a point and a straight line. We try to develop the idea in the work. Also we suggest a new geometric object - a polyangle and connected with it extensions of conform mappings and analytical functions.

Symmetric Finsler spaces

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Symmetric spaces were first defined and studied by E. Cartan, and they have been generalized to s-manifolds by A.J.Ledger and M. Obata and to reflection spaces by O. Loos. Recently we have defined symmetric Finsler spaces as Finsler manifolds having an action “.” satisfying four properties. In this talk I am going to reduce some of these properties and prove that they are still homogeneous and reductive.

On Finsler spaces with constant Ricci scalar curvature

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In this paper we consider some (α, β) -metrics such as generalized Kropina, Matsumoto metrics, and obtain the necessary and sufficient conditions for them that be Einstein metrics while β is constant killing form. Then we prove with this assumption, the mention Einstein metrics must be Ricci flat or Riemannian. we, also, prove that Einstein Berwald metrics are constant Ricci scalar metrics.

Theorem. *Let (M, F) be a Finsler space of Matsumoto metric F with constant killing form β , then F is Einstein if and only if it is Riemannian Einstein metric.*

Theorem. *An Einstein generalized Kropina metric with constant killing form β must be Riemannian.*

Projectively Related Einstein Finsler Spaces

N. Sadeghzadeh and B. Rezaei

The main objective of this paper is to find the necessary and sufficient condition of a given Finsler metric to be Einstein in order to classify the Einstein Finsler metrics on a compact manifold. The considered Einstein Finsler metric

in the study describes all different kinds of Einstein metrics which are pointwise projective to the given one. The study is resulted in the following theorem that needs the proof of three prepositions. Let F be a Finsler metric projectively related to \bar{F} where \bar{F} is an Einstein non-isotropic Finsler metric, then F is Einstein if and only if is a constant coefficient of \bar{F} . ($n > 2$)

Isometries between Finsler spaces

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We present some results concerning isometries between Finsler spaces. One of them shows that a mapping between two Finsler spaces is an isometry iff it keeps (2-dimensional) area and (Minkowski) angle. This is an extension of a well known theorem of Riemannian spaces to Finsler spaces. Another one reduces the isometry of two Finsler spaces to that of Riemannian spaces. In relation with this we present a decomposition theorem of Finsler spaces, which yields necessary conditions for Finsler isometries, and for the existence of 1-parameter family of motions in Finsler spaces.

Throughout of these we use short, purely geometric considerations.

Geometry of two-step nilpotent groups with left invariant Finsler metrics

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We begin a systematic study of these spaces following Eberlein's comprehensive study in the Riemannian case. In particular, for some special groups (including Heisenberg groups) we give an explicit form for the Chern-Rund connection, study the curvature and the flag curvature, and derive the equations of relative geodesics.

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On Berwald and Wagner spaces

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Berwald and Wagner spaces are two important classes of spaces in Finsler geometry. They are closely related to each other via the conformal change of the metric. After discussing the basic definitions and the elements of the theory we present general methods to construct examples of them.

On rigidity properties for weakly Landsberg manifolds

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In this talk we will study the rigidity properties for weakly Landsberg manifolds and prove that a closed weakly Landsberg manifold with negative flag curvature must be Riemannian.

Nonlinear dynamical systems and KCC-theory

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In this study, we discuss that nonlinear dynamical systems can be uniquely investigated by the geometric theory (KCC-theory). Then, we apply the KCC-theory to a geodynamical system called the Rikitake system and show a relation between the geometrical objects and the behavior of magnetic field.

We consider a system of second order differential equations:

$$\frac{d^2x^i}{d\tau^2} + 2G^i \left(x^j, \frac{dx^j}{d\tau}, \tau \right) = 0, \quad i, j = 1, \dots, n, \quad (1)$$

where x^i is a coordinate of the configuration space, τ is a certain parameter such as a time t and G^i is a smooth function. The geometric theory of the system of second order differential equations is named by the KCC-theory (Kosambi

[5], Cartan [2], Chern [3]) and the geometric invariants of the system (1) can be obtained from this theory [1].

The equation (1) can describe various nonlinear dynamical systems. For example, in biology, the Volterra-Hamilton system belongs to the equation (1) and has been geometrically studied using the KCC-theory [1]. In chemistry, the Belousov-Zhabotinskii reaction model is described by the equation (1). Moreover, in the Earth sciences, the above equation (1) can describe some geophysical phenomena. For the Lorenz model in meteorology, the equation of motion is equivalent to the geodesic equation (1) in Finsler geometry [4]. In geodynamo theory, a unified electro-mechanical system (the Rikitake system) is expressed by the equation (1) and describes the chaotic behavior of magnetic field [6]. This Rikitake system has been geometrized based on the KCC-theory [7]. Therefore, the nonlinear dynamical systems denoted by (1) can connect with the geometrical concept via the KCC-theory and Finsler geometry.

We show the geometrical description of the Rikitake system based on the KCC-theory. The connection represents the electro-mechanical interactions. Then, the five geometric invariants can be obtained. The second invariant (curvature tensor) determines the stability of the system. The third invariant (torsion tensor) can be represented by mutual-inductances as a result of electro-mechanical interactions which cause the aperiodic magnetic reversal. This torsion tensor expresses an electric current discrepancy given by a projection from the mechanical field to the electrical field. Hence, the chaotic behavior of the magnetic field can be investigated by the geometrical invariants.

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