Erratum

to the paper "Some results on the geometry of tangent bundle of Finsler manifolds", Publ. Math. Debrecen (2007), 185–193

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The formula (4.3) is incorrect. It should be the following:

$$\begin{split} \widetilde{R}\left(\frac{\partial}{\partial y^{i}},\frac{\partial}{\partial y^{j}}\right)\frac{\partial}{\partial y^{k}} &= \widetilde{\nabla}_{\frac{\partial}{\partial y^{i}}}\widetilde{\nabla}_{\frac{\partial}{\partial y^{j}}}\frac{\partial}{\partial y^{k}} - \widetilde{\nabla}_{\frac{\partial}{\partial y^{j}}}\widetilde{\nabla}_{\frac{\partial}{\partial y^{i}}}\frac{\partial}{\partial y^{k}} - \widetilde{\nabla}_{\left[\frac{\partial}{\partial y^{i}},\frac{\partial}{\partial y^{j}}\right]}\frac{\partial}{\partial y^{k}} \\ &= \left(\frac{\partial L^{l}_{jk}}{\partial y^{i}} - \frac{\partial L^{l}_{ik}}{\partial y^{j}} + L^{s}_{jk}C^{l}_{si} + C^{s}_{jk}L^{l}_{si} - L^{s}_{ik}C^{l}_{sj} - C^{s}_{ik}L^{l}_{sj} \right. \\ &+ \left. \frac{1}{2}L^{t}_{jk}y^{s}R_{sit}^{l} - \frac{1}{2}L^{t}_{ik}y^{s}R_{sjt}^{l}\right)\frac{\delta}{\delta x^{l}} \\ &+ \left(L^{s}_{ik}L^{l}_{js} - L^{s}_{jk}L^{l}_{si} + C^{s}_{ik}C^{l}_{js} - C^{s}_{jk}C^{l}_{si}\right)\frac{\partial}{\partial y^{l}}, \end{split} \tag{4.3}$$

here the last two terms $C_{ik}^s C_{js}^l - C_{jk}^s C_{si}^l$ were missed in the original paper. Fortunately, we need only to make a minor modification. Assume that $\widetilde{\nabla} \widetilde{R} = 0$, then by (2.3), (4.4) and Lemma 2.1 we have

$$\begin{split} 0 &= y^j \left(\widetilde{\nabla}_{\eta} \widetilde{R} \right) \left(\frac{\partial}{\partial y^i}, \frac{\partial}{\partial y^j} \right) \frac{\partial}{\partial y^k} = y^j \widetilde{\nabla}_{\eta} \left(\widetilde{R} \left(\frac{\partial}{\partial y^i}, \frac{\partial}{\partial y^j} \right) \frac{\partial}{\partial y^k} \right) \\ &= -y^j \frac{\partial L^l_{jk}}{\partial y^i} \frac{\delta}{\delta x^l} = L^l_{ik} \frac{\delta}{\delta x^l}, \end{split}$$

and hence, $L_{jk}^{i} = 0$, which together with (4.3) yields

$$\widetilde{R}\left(\frac{\partial}{\partial y^i},\frac{\partial}{\partial y^j}\right)\frac{\partial}{\partial y^k} = \left(C^s_{ik}C^l_{js} - C^s_{jk}C^l_{si}\right)\frac{\partial}{\partial y^l}.$$

The author is indebted to Professor Aurel Bejancu for pointing out the mistake.

Consequently,

$$0 = \left(\widetilde{\nabla}_{\eta}\widetilde{R}\right) \left(\frac{\partial}{\partial y^{i}}, \frac{\partial}{\partial y^{j}}\right) \frac{\partial}{\partial y^{k}} = -2 \left(C^{s}_{ik}C^{l}_{js} - C^{s}_{jk}C^{l}_{si}\right) \frac{\partial}{\partial y^{l}},$$

and we again arrive at $\widetilde{R}\left(\frac{\partial}{\partial y^i}, \frac{\partial}{\partial y^j}\right) \frac{\partial}{\partial y^k} = 0$.

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