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Title: Two terms with known prime divisors adding to a power

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Let c be a positive odd integer, and R a set of n primes coprime with c. We consider equations $X + Y = c^z$ in three integer unknowns X, Y, z, where $z > 0$, $Y > X > 0$, and the primes dividing XY are precisely those in R. We consider N, the number of solutions of such an equation. Given a solution (X, Y, z), let D be the least positive integer such that $(XY/D)^{1/2}$ is an integer. Further, let ω be the number of distinct primes dividing c. Standard elementary approaches use an upper bound of 2^n for the number of possible D, and an upper bound of $2^{\omega-1}$ for the number of ideal factorizations of c in the field $\mathbb{Q}(\sqrt{-D})$ which can correspond (in a standard designated way) to a solution in which $(XY/D)^{1/2} \in \mathbb{Z}$, and obtain $N \leq 2^{n+\omega-1}$. Here we improve this by finding an inverse proportionality relationship between a bound on the number of D which can occur in solutions and a bound (independent of D) on the number of ideal factorizations of c which can correspond to solutions for a given D. We obtain $N \leq 2^{n-1} + 1$. The bound is precise for $n < 4$: there are several cases with exactly $2^{n-1} + 1$ solutions.

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