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Generic submanifolds of a trans-Sasakian manifold

By M. HASAN SHAHID (New Delhi) and ION MIHAI (Bucharest)

In [C2] B.Y. Chen introduced the notion of a *generic submanifold* of a *Kaehler manifold* and obtained interesting properties. The class of *generic submanifolds* includes *complex*, *totally real*, *slant* and *CR*-submanifolds. Generic submanifolds of Sasakian manifolds and of framed f-manifolds have been studied by the present authors (see [HA], [M2]), P. Verheyen [V], etc. In [O], J.A. Oubina introduced a new class of almost contact metric manifolds and called them *trans-Sasakian* manifolds. This class contains both α-*Sasakian* and β-*Kenmotsu* manifolds (see [OR]).

The aim of the present paper is to study *generic* submanifolds of *trans-Sasakian* manifolds. Certain submanifolds of a β-*Kenmotsu* manifold were investigated in [MMR].

1. Preliminaries

Let $\bar{M}$ be a $(2n+1)$-dimensional *almost contact metric* manifold with an *almost contact metric* structure (ϕ,ξ,η,g). Then we have [B]

$$\phi^2=-I+\eta\otimes\xi,\quad \phi\xi=0,\quad \eta\circ\phi=0,\quad \eta(\xi)=1, \tag{1.1}$$

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y),\quad \eta(X)=g(X,\xi), \tag{1.2}$$

for any vector fields X,Y on $\bar{M}$, where I denotes the identity transformation on $T\bar{M}$.

An almost contact structure (ϕ,ξ,η) is said to be *normal* if the almost complex structure J on $\bar{M}\times\mathbb{R}$ given by

$$J\left(X,f\frac{d}{dt}\right)=\left(\phi X-f\xi,\eta(X)\frac{d}{dt}\right), \tag{1.3}$$

where f is a C^∞ function on $\bar{M}\times\mathbb{R}$, is integrable, or equivalently $[\phi,\phi]+2d\eta\otimes\xi=0$, where $[\phi,\phi]$ denotes the Nijenhuis tensor of ϕ.

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In the classification of A. Gray and L.M. Hervella [GH] of almost *Hermitian* manifolds, there appears a class of *Hermitian* manifolds called $\mathcal{W}_4$, which contains *locally conformal Kaehler* manifolds. An almost contact metric manifold $\bar{M}$ is called *trans-Sasakian* if $(\bar{M}\times\mathbb{R}, J, G)$ belongs to the class $\mathcal{W}_4$, where J is the almost complex structure on $\bar{M}\times\mathbb{R}$ defined by (1.3) and G is the product Riemannian metric on $\bar{M}\times\mathbb{R}$. This may be expressed by the condition (see [BO])

$$(\bar{\nabla}_X\phi)Y = \alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \eta(Y)\phi X\}, \tag{1.4}$$

for some functions α and β on $\bar{M}$, and we say that the trans-Sasakian structure is of type (α,β). In particular, it is normal. From (1.4), one easily obtains

$$\bar{\nabla}_X\xi = -\alpha\phi X + \beta(X - \eta(X)\xi). \tag{1.5}$$

In the following, by a trans-Sasakian manifold we always mean a trans-Sasakian manifold of type (α,β).

Let M be an n-dimensional isometrically immersed submanifold of $\bar{M}$, tangent to ξ. Let g be the metric tensor field on $\bar{M}$ as well as the induced metric on M. We denote by ∇ the Riemannian connection with respect to g on M. The Gauss and Weingarten formulae are respectively given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{1.6}$$

$$\bar{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \tag{1.7}$$

where $X, Y \in \Gamma(TM)$ and $N \in \Gamma(T^{\perp}M)$. We recall that the second fundamental form h and the shape operator A_N are related by

$$g(A_N X, Y) = g(h(X,Y), N).$$

Definition. Let M be a submanifold of an almost contact metric manifold $\bar{M}$. If the maximal invariant subspaces by ϕ, orthogonal to ξ in T_xM,

$$\mathcal{D}_x = T_xM \cap \phi T_xM, \quad x \in M$$

define a differentiable subbundle of TM (i.e. the dimension of $\mathcal{D}_x$ is constant along M), then M is called a *generic* submanifold of $\bar{M}$.

For any vector field X tangent to M, we put

$$\phi X = PX + QX, \tag{1.8}$$

where PX and QX denote the tangential and normal components of ϕX respectively.

For any vector field N normal to M, we put

$$\phi N = BN + CN, \tag{1.9}$$

where BN and CN denote the tangential and normal components of ϕN respectively.

We call $\mathcal{D}$ the holomorphic distribution and the subbundle $\mathcal{D}^{\perp}$ orthogonal to $\mathcal{D} \oplus \{\xi\}$ in TM the purely real distribution. They satisfy the following relations:

$$\mathcal{D}_x \perp \mathcal{D}_x^{\perp}, \quad \mathcal{D}_x^{\perp} \cap \phi\mathcal{D}_x^{\perp} = \{0\}, \quad P\mathcal{D}_x = \mathcal{D}_x, \quad P\mathcal{D}_x^{\perp} \subset \mathcal{D}_x^{\perp}. \tag{1.10}$$

If the purely real distribution is an anti-invariant subbundle by ϕ, i.e. $\phi(\mathcal{D}_x^{\perp}) \subset T_x^{\perp}M$, $\forall x \in M$, then M is called a *CR-submanifold* of $\bar{M}$ (see, for instance, [HA], [M1]).

Let ν_x be the maximal invariant vector subspace of $T_x^{\perp}M$, i.e.

$$\nu_x = T_x^{\perp}M \cap \phi(T_x^{\perp}M) \quad ;$$

then ν_x defines a differentiable subbundle of $T^{\perp}M$, satisfying

$$T^{\perp}M = Q\mathcal{D}^{\perp} \oplus \nu, \quad B(T^{\perp}M) \subset \mathcal{D}^{\perp}, \quad Q\mathcal{D}^{\perp} \perp \nu. \tag{1.11}$$

Examples. Let $\mathbb{E}^{2n+1} = \mathbb{C}^n \times \mathbb{R}$ be the $(2n+1)$-dimensional Euclidean space endowed with the standard almost contact metric structure (ϕ, ξ, η, g), defined by (see [B])

$$\phi(x^1, ..., x^{2n}, z) = (-x^2, x^1, ..., -x^{2n}, x^{2n-1}, 0),$$
$$\eta = dz, \quad \xi = \frac{\partial}{\partial z},$$

and $1 \leq h \leq n$. The product $M_1 \times M_2 \times \mathbb{R}$, where M_1 is a complex submanifold of $\mathbb{C}^h$ and M_2 a slant submanifold of $\mathbb{C}^{n-h}$, is a generic submanifold of $\mathbb{E}^{2n+1}$ (for definition and examples of *slant* submanifolds see B.Y. Chen's book [C4]).

2. Integrability of distributions

In this section, we shall study the integrability conditions for the distributions on a generic submanifold of a trans-Sasakian manifold.

Proposition 2.1. *Let $\bar{M}$ be a trans-Sasakian manifold with $\alpha \neq 0$ and M a generic submanifold of $\bar{M}$. Then the distribution $\mathcal{D}$ is never integrable.*

Proof. Let X be a non-zero vector field belonging to $\mathcal{D}$. Then from (1.5) it follows that

$$g([X, \phi X], \xi) = 2\alpha g(X, X) \neq 0;$$

thus $\mathcal{D}$ cannot be integrable.

Theorem 2.2. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. Then the following assertions are equivalent to each other:*

i) the distribution $\mathcal{D} \oplus \{\xi\}$ is integrable;

ii) $h(\phi X, Y) = h(X, \phi Y)$ for any $X, Y \in \Gamma(\mathcal{D})$, i.e. ϕ is self adjoint on $\mathcal{D}$ with respect to the second fundamental form h;

iii) $g(h(\phi X, Y), \phi Z) = g(h(X, \phi Y), \phi Z)$, for any $X, Y \in \Gamma(\mathcal{D})$ and $Z \in \Gamma(\mathcal{D}^\perp)$.

Proof. From (1.4) and using Gauss formula, we get

$$\begin{aligned} h(X, \phi Y) = {} & \phi \nabla_X Y + \phi h(X, Y) - \nabla_X \phi Y \\ & + \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \eta(Y)\phi X\}, \end{aligned}$$

for any $X, Y \in \Gamma(\mathcal{D})$.

Thus we have

$$h(X, \phi Y) - h(\phi X, Y) = \phi[X, Y] - \nabla_X \phi Y + \nabla_Y \phi X + 2\beta g(\phi X, Y)\xi.$$

Taking the normal part of the right term, we find

$$h(X, \phi Y) - h(\phi X, Y) = Q[X, Y].$$

Therefore $\mathcal{D} \oplus \{\xi\}$ is involutive if and only if

$$h(\phi X, Y) = h(X, \phi Y), \quad \forall X, Y \in \Gamma(\mathcal{D}).$$

The other equivalences are obvious.

Corollary 2.3. *Let $\bar{M}$ be a β-Kenmotsu manifold and M a generic submanifold. Then the distribution $\mathcal{D}$ is integrable if and only if the above assertions i)-iii) hold good.*

Next, we concentrate on the integrability of the purely real distribution.

Theorem 2.4. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. Then the following assertions are equivalent to each other:*

i) the distribution $\mathcal{D}^\perp \oplus \{\xi\}$ is integrable;

ii) $A_{QW}Z - A_{QZ}W + \nabla_W PZ - \nabla_Z PW \in \Gamma(\mathcal{D}^\perp)$, for any $Z, W \in \Gamma(\mathcal{D}^\perp)$.

Proof. For any vector fields $Z, W \in \Gamma(\mathcal{D}^\perp)$ and $X \in \Gamma(\mathcal{D})$, we have

$$\begin{aligned} g([Z, W], \phi X) = {} & -g(\bar{\nabla}_Z \phi W, X) + g(\bar{\nabla}_W \phi Z, X) = \\ & -g(\nabla_Z PW - A_{QW}Z - \nabla_W PZ + A_{QZ}W, X). \end{aligned}$$

Using Frobenius theorem, it follows that $\mathcal{D}^\perp \oplus \{\xi\}$ is integrable if and only if ii) holds good.

Lemma 2.5. *Let M be a CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then*

$$A_{QZ}W = A_{QW}Z,$$

for any $Z, W \in \Gamma(\mathcal{D}^\perp)$.

Proof. For $Z, W \in \Gamma(\mathcal{D}^\perp)$ and $Y \in \Gamma(TM)$, using (1.4), (1.6) and (1.7), we have

$$g(A_{QW}Z, Y) = g(h(Y,Z), \phi W) = g(\bar{\nabla}_Y Z, \phi W) = -g(\phi\bar{\nabla}_Y Z, W) = \\ -g(\bar{\nabla}_Y \phi Z, W) = g(A_{QZ}Y, W) = g(A_{QZ}W, Y),$$

which achieves the proof.

Proposition 2.6. *Let $\bar{M}$ be a trans-Sasakian manifold with $\alpha \neq 0$ and M a generic submanifold of $\bar{M}$. Then the distribution $\mathcal{D}^\perp$ is integrable if and only if M is a CR-submanifold.*

Proof. Let $Z, W \in \Gamma(\mathcal{D}^\perp)$. If $\mathcal{D}^\perp$ is integrable, we have

$$0 = g([Z,W], \xi) = 2\alpha g(\phi Z, W),$$

i.e. M is a CR-submanifold.

Conversely, if M is a CR-submanifold, then by Theorem 2.4 and Lemma 2.5 it follows that $\mathcal{D}^\perp$ is integrable (see also [H], [M2], [V]).

3. Generic submanifolds with parallel canonical structures

Let P, C, Q and B be the endomorphisms and the vector bundle-valued 1-forms defined by (1.8) and (1.9) respectively. Now, let us define the covariant differentiations of P, Q, B and C as follows:

$$(\bar{\nabla}_X P)Y = \nabla_X PY - P\nabla_X Y, \tag{3.1}$$

$$(\bar{\nabla}_X Q)Y = \nabla^\perp_X QY - Q\nabla_X Y, \tag{3.2}$$

$$(\bar{\nabla}_X B)N = \nabla_X BN - B\nabla^\perp_X N, \tag{3.3}$$

$$(\bar{\nabla}_X C)N = \nabla^\perp_X CN - C\nabla^\perp_X N, \tag{3.4}$$

for any vector fields X and Y tangent to M and any vector field N normal to M.

Definition. The endomorphism P (resp. the endomorphism C, the 1-forms Q and B) is called *parallel* if $\bar{\nabla} P = 0$ (resp. $\bar{\nabla} C = 0$, $\bar{\nabla} Q = 0$ and $\bar{\nabla} B = 0$).

Now, from (1.4) and using (1.6)–(1.8) we have

$$\begin{gathered}\nabla_X PY + h(X,PY) - A_{QY}X + \nabla^{\perp}_X QY - P\nabla_X Y - \\ -Q\nabla_X Y - Bh(X,Y) - Ch(X,Y) \\ = \alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(PX,Y)\xi - \eta(X)(PX+QX)\},\end{gathered}$$

for all $X, Y \in \Gamma(TM)$.

Comparing tangential and normal components respectively, we get

$$\begin{gathered}\nabla_X PY - P\nabla_X Y = (\bar{\nabla}_X P)Y = A_{QY}X + Bh(X,Y) \\ +\alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(PX,Y)\xi - \eta(Y)PX\},\end{gathered} \tag{3.5}$$

$$\begin{gathered}\nabla^{\perp}_X QY - Q\nabla_X Y = (\bar{\nabla}_X Q)Y \\ = Ch(X,Y) - h(X,PY) - \beta\eta(Y)QX,\end{gathered} \tag{3.6}$$

for any $X, Y \in \Gamma(TM)$.

Analogously, we find

$$\begin{gathered}\nabla_X BN - B\nabla^{\perp}_X N = (\bar{\nabla}_X B)N \\ = A_{CN}X - PA_N X + \beta g(QX,N)\xi,\end{gathered} \tag{3.7}$$

$$\nabla^{\perp}_X CN - C\nabla^{\perp}_X N = (\bar{\nabla}_X C)N = -h(X,BN) - QA_N X. \tag{3.8}$$

Lemma 3.1. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. Then the endomorphism P is parallel if and only if*

$$A_{QX}Y - A_{QY}X = \alpha\{\eta(X)Y - \eta(Y)X\} + \beta\{\eta(Y)PX - \eta(X)PY\},$$

for any vector fields X, Y tangent to M.

Proof. From (3.5) we have

$$\begin{gathered}g((\bar{\nabla}_X P)Y, Z) = g(A_{QY}X, Z) + g(Bh(X,Y), Z) \\ +\alpha\{g(X,Y)\eta(Z) - g(X,Z)\eta(Y)\} + \beta\{g(PX,Y)\eta(Z) - g(PX,Z)\eta(Y)\} \\ = g(A_{QY}X, Z) - g(A_{QZ}X, Y) + \alpha\{g(X,Y)\eta(Z) - g(X,Z)\eta(Y)\} \\ +\beta\{g(PX,Y)\eta(Z) - g(PX,Z)\eta(Y)\},\end{gathered}$$

which proves our assertion.

Next, we have the following

Proposition 3.2. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. If P is parallel, then:*

i) the holomorphic distribution $\mathcal{D} \oplus \{\xi\}$ is integrable;

ii) $A_{QU}X = \alpha\{\eta(U)X - \eta(X)U\} + \beta\{\eta(X)PU - \eta(U)PX\}$, for any $X \in \Gamma(\mathcal{D})$ and $U \in \Gamma(TM)$.

Proof. Assume that P is parallel, i.e. $\bar{\nabla}P = 0$; then Lemma 3.1 gives

$$A_{QU}X = \alpha\eta(U)X - \beta\eta(U)PX,$$

for any $X \in \Gamma(\mathcal{D})$ and $U \in \Gamma(TM)$.

For $U = Z \in \Gamma(\mathcal{D}^\perp), X, Y \in \Gamma(\mathcal{D})$, and using $P\mathcal{D}^\perp \subset \mathcal{D}^\perp$ we get

$$g(A_{QZ}X, Y) = 0, \quad \text{i.e.} \quad g(h(X,Y), QZ) = 0.$$

Proposition 3.3. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. Then Q is parallel if and only if B is parallel.*

Proof. Suppose that B is parallel, i.e. $\bar{\nabla}B = 0$. Then from (3.7) it follows that

$$g(A_{CN}X, Y) = g(PA_NX, Y) - \beta g(QX, N)g(Y, \xi),$$

for any vector fields X, Y tangent to M and N normal to M.

Hence we have

$$g(Ch(X,Y), N) = g(h(X, PY), N) + \beta g(QX, N)\eta(Y),$$

which is equivalent to

$$Ch(X,Y) = h(X, PY) + \beta\eta(Y)QX,$$

i.e. $\bar{\nabla}Q = 0$.

The proof for the converse statement is similar.

4. Geometry of leaves on generic submanifolds

From (3.5) and (3.6) we have

$$\begin{aligned} P\nabla_X Z = &-A_{QZ}X - Bh(X,Z) + \alpha\{\eta(Z)X - g(X,Z)\xi\} \\ &+\beta\{\eta(Z)PX - g(PX,Z)\xi\} + \nabla_X PZ \end{aligned} \tag{4.1}$$

and

$$Q\nabla_X Z = \nabla^\perp_X QZ - Ch(X,Z) - \beta\eta(Z)QX - h(X,PZ), \tag{4.2}$$

for any $X \in \Gamma(TM)$ and $Z \in \Gamma(\mathcal{D}^\perp)$.

Now we prove the following

Proposition 4.1. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. Then the distribution $\mathcal{D} \oplus \{\xi\}$ is integrable and its leaves are totally geodesic in M if and only if*

$$g(h(\mathcal{D}, \mathcal{D}), Q\mathcal{D}^\perp) = 0. \tag{4.3}$$

Proof. Let $X, Y \in \Gamma(\mathcal{D})$ and $Z \in \Gamma(\mathcal{D}^\perp)$. If the distribution $\mathcal{D} \oplus \{\xi\}$ is integrable and its leaves are totally geodesic in M, then $\nabla_X \phi Y \in \Gamma(\mathcal{D} \oplus \{\xi\})$.

By (4.1) and using $B(T^\perp M) \subset \mathcal{D}^\perp$, we have

$$0 = g(\nabla_X \phi Y, Z) = -g(\nabla_X Z, \phi Y) = g(P\nabla_X Z, Y) =$$
$$-g(A_{QZ}X, Y) - g(Bh(X, Z), Y) + g(\nabla_X PZ, Y) = -g(h(X, Y), QZ).$$

Conversely, suppose that (4.3) holds good. Then the distribution $\mathcal{D} \oplus \{\xi\}$ is integrable by virtue of Theorem 2.2. Now, using (1.4) we get

$$0 = g(h(X, \phi Y), QZ) = g(\bar{\nabla}_X \phi Y, QZ) = g(\phi \bar{\nabla}_X Y, QZ) = g(\nabla_X Y, Z),$$

for any $X, Y \in \Gamma(\mathcal{D}), Z \in \Gamma(\mathcal{D}^\perp)$.

Thus $\nabla_X Y \in \Gamma(\mathcal{D})$ for any $X, Y \in \Gamma(\mathcal{D})$ and each leaf of $\mathcal{D}$ is totally geodesic in M, which completes the proof.

Proposition 4.2. *Let M be a generic submanifold of a trans-Sasakian manifold $\bar{M}$. If the distribution $\mathcal{D}^\perp$ is integrable, then its leaves are totally geodesic in M if and only if*

$$g(h(X, W), QZ) = 0,$$

for any $Y \in \Gamma(\mathcal{D})$ and $Z, W \in \Gamma(\mathcal{D}^\perp)$.

Proof. From (3.5) and (3.6) we get

$$P\nabla_X Z = -A_{QZ}X - Bh(X, Z) - \alpha g(X, Z)\xi - \beta g(PX, Z)\xi \tag{4.5}$$

and

$$Q\nabla_X Z = \nabla_X^\perp QZ - Ch(X, Z), \tag{4.6}$$

for any $X \in \Gamma(TM)$ and $Z \in \Gamma(\mathcal{D}^\perp)$.

Putting $X = W \in \Gamma(\mathcal{D}^\perp)$ in (4.5), we have

$$P\nabla_X Z = -A_{QZ}W - Bh(Z, W) - \alpha g(Z, W)\xi - \beta g(Z, PW)\xi.$$

Taking the inner product with $Y \in \Gamma(\mathcal{D})$, we obtain

$$g(\nabla_W Z, PY) = g(h(Y, W), QZ), \quad \forall Z, W \in \Gamma(\mathcal{D}^\perp), Y \in \Gamma(\mathcal{D}).$$

The proof is complete.

5. The CR-structure of a generic submanifold

Each generic submanifold of a Kaehler manifold carries a canonical *Cauchy-Riemann* (abr. CR) structure in the sense of S. Greenfield [G] (see [Op]). This result was extended to Sasakian and β-Kenmotsu manifolds ([V], [MMR]). It is also true in the case under consideration.

Recall the definition of a Cauchy-Riemann structure [G].

A complex distribution $\mathcal{H}$ on M (i.e. $\mathcal{H} \subset TM \otimes_{\mathbb{R}} \mathbb{C}$) is said to define a *Cauchy-Riemann* structure if it satisfies the following conditions:

i) $\mathcal{H} \cap \bar{\mathcal{H}} = \{0\}$, where $\bar{\mathcal{H}}$ is the conjugated distribution of $\mathcal{H}$;

ii) $\mathcal{H}$ is involutive, i.e. for any $A, B \in \Gamma(\mathcal{H})$, $[A, B] \in \Gamma(\mathcal{H})$.

Theorem 5.1. *Each generic submanifold M of a trans-Sasakian manifold is a Cauchy-Riemann manifold.*

Proof. Let $l : TM \to \mathcal{D}$ and $m : TM \to \mathcal{D}^{\perp}$ be the projection operators. Then each vector field X can be expressed by $X = lX + mX + \eta(X)\xi$.

We put $\mathcal{H} = \{X - i\phi X | X \in \Gamma(\mathcal{D})\}$.

Let $A, B \in \Gamma(\mathcal{H})$; then $A = X - i\phi X$, $B = Y - i\phi Y$, for certain $X, Y \in \Gamma(\mathcal{D})$.

$\bar{M}$ being normal, we have $[\phi, \phi](X, Y) + 2d\eta(X, Y)\xi = 0$. Then we get

$$[\phi X, \phi Y] - [X, Y] - \phi l\{[\phi X, Y] + [X, \phi Y]\} = 0,$$
$$m\{[\phi X, Y] + [X, \phi Y]\} = 0.$$

Replacing X by ϕX, we obtain

$$[\phi X, \phi Y] - [X, Y] \in \Gamma(\mathcal{D}).$$

On the other hand, we may write

$$\begin{aligned}[A, B] &= [X, Y] - [\phi X, \phi Y] - i[\phi X, Y] - i[X, \phi Y] \\ &= [X, Y] - [\phi X, \phi Y] - i\phi\{[X, Y] - [\phi X, \phi Y]\} \in \Gamma(\mathcal{H})\end{aligned}$$

and the proof is complete.

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M. HASAN SHAHID
DEPARTMENT OF MATHEMATICS
FACULTY OF NATURAL SCIENCES
JAMIA MILLIA ISLAMIA
NEW DELHI–25
INDIA

ION MIHAI
FACULTY OF MATHEMATICS
STR. ACADEMIEI 14
70109 BUCHAREST
ROMANIA

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